ASP Final Project: Adaptive Beamforming

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1. Details of the beamforming

At first, we make some assumptions:

1. There is only one source,
2. The DOA of the source is ,
3. Uniform linear array with N isotropic antennas and inter-element spacing ,
4. The source waveform is , which the complex amplitude A satisfies and ,
5. The noise term satisfies and **.**
6. The beamformer with uniform weights

At this model, we assume that the DOA =0, so every antenna will receive source signals with the same phase. The equation of the data model is denoted as:

Where is known as steering vector, written as:

We could apply uniform weightings , so the beamformer output is:

By doing this, we could recover the source signal while the noise is decreased by a factor 1/N.

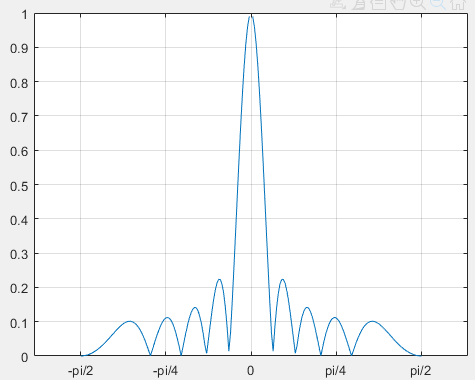
The SNR then becomes to:

This model will get the best performance when the source . If , the SNR will decrease. For example, let’s set , then the input is rewritten as:

The output becomes to:

is an important factor for SNR. Below show the expression of .

From the figure below, we could observe that get its maximum at rad =0, and at some specified angle, though.



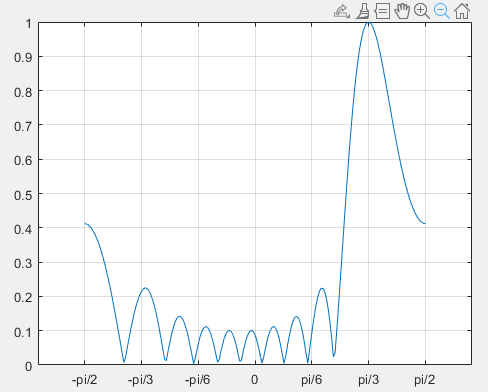
Figure(a): plot of with radius changing. Setting N = 10

1. The beamformer with array steering

Since uniform weightings only works well at degree=0, we want to derive a method for specified angle of DOA. Therefore, at this model, we assume that the DOA . And we adjust the weight vector so that the signal waveform is kept when it near , and is rejected when it is far from . The new weight vector is denoted as:

So we could rewrite as:

is still an important factor of SNR, and from the figure below, we can observe that will get its maximum at . Therefore, the beamformer of with array steering performs better when the source signal comes from the same direction . The difficulty is how to know the DOA.



Figure(b): Plot of with changing. Set N =10 and

1. The MVDR beamformer

Since the output is denoted as:

we could have minimum mean square error with the distortionless constraint:

But the correlation of noise is unavailable. We can derive it from the power of output with the constraint :

If the correlation of noise and desired signal is **0**, then shares the same weight vector with . Thus, the optimization problem becomes to:

Assuming that is WSS, so the correlation matrix .

Solving the optimization function, we could get:

1. The LCMV beamformer

Just like MVDR beamformer, LCMV beamformer is an optimization with linear constraint. The optimization problem is:

If , then LCMV is equivalent to MVDR.

Solving the optimization problem, we get:

When the collected data contains desired signal and interference, and their directions are different, we could apply LCMV by selecting the following constraints:

1. Design an algorithm to denoise      and over the time index t. The denoised results are denoted by and .